Wilcoxon Rank Sum Test

Wilcoxon Rank Sum Test Procedure

1. Assign Ranks, R_i, to the $n_1 + n_2$ Sample Observations

   If Unequal Sample Sizes, Let n_1 Refer to Smaller-Sized Sample

   Smallest Value = 1

2. Sum the Ranks, T_i, for Each Sample

Test Statistic Is T_A (Smallest Sample)

Null hypothesis: both samples come from the same underlying distribution

Distribution of T is not quite as simple as binomial, but it can be computed

Wilcoxon Rank Sum Test Example

- You're a production planner. You want to see if the operating rates for 2 factories is the same. For factory 1, the rates (% of capacity) are **71**, **82**, **77**, **92**, **88**. For factory 2, the rates are **85**, **82**, **94** & **97**. Do the factory rates have the same **probability distributions**?

Wilcoxon Rank Sum Test Solution

- **H_0:**
- **H_a:**
- **n_1 = n_2 =**
- **Critical Value(s):**

Test Statistic:

Decision:

Σ Ranks

Wilcoxon Rank Sum Test Solution

- **H_0: Identical Distrib.**
- **H_a: Shifted Left or Right**
- **n_1 = n_2 =**
- **Critical Value(s):**

Test Statistic:

Decision:

Σ Ranks

Wilcoxon Rank Sum Test Solution

- **H_0: Identical Distrib.**
- **H_a: Shifted Left or Right**
- **$n_1 = 5$ $n_2 = 4$**
- **Critical Value(s):**

Test Statistic:

Decision:

Σ Ranks

Wilcoxon Rank Sum Table 12 (Rosner) (Portion)

α= .05 two-tailed

a. $\alpha = .025$ one-tailed; $\alpha = .05$ two-tailed

n_2 \ n_1	3		4		5		6		7		8	
	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U
3	5	16	6	18	6	21	7	23	7	26	8	28
4	6	18	11	25	12	28	12	32	13	35	14	38
5	6	21	12	28	18	37	19	41	20	45	21	49
6	7	23	12	32	19	41	26	52	28	56	29	61
7	7	26	13	35	20	45	28	56	37	68	39	73
8	8	28	14	38	21	49	29	61	39	73	49	87
9	8	31	15	41	22	53	31	65	41	78	51	93
10	9	33	16	44	24	56	32	70	43	83	54	98

Wilcoxon Rank Sum Test Solution

- **H_0: Identical Distrib.**
- **H_a: Shifted Left or Right**
- **$n_1 = 5$ $n_2 = 4$**
- **Critical Value(s):**

Test Statistic:

Decision:

Reject	Do Not Reject	Reject

12 28 Σ Ranks

Wilcoxon Rank Sum Test Computation Table

Factory 1		Factory 2	
Rate	**Rank**	**Rate**	**Rank**
Rank Sum			

Wilcoxon Rank Sum Test Computation Table

Factory 1		Factory 2	
Rate	Rank	Rate	Rank
71		85	
82		82	
77		94	
92		97	
88		...	...
Rank Sum			

Wilcoxon Rank Sum Test Computation Table

Factory 1		Factory 2	
Rate	Rank	Rate	Rank
71	1	85	
82		82	
77		94	
92		97	
88		...	...
Rank Sum			

Wilcoxon Rank Sum Test Computation Table

Factory 1		Factory 2	
Rate	Rank	Rate	Rank
71	1	85	
82		82	
77	2	94	
92		97	
88		...	...
Rank Sum			

Wilcoxon Rank Sum Test Computation Table

Factory 1		Factory 2	
Rate	Rank	Rate	Rank
71	1	85	
82	3	82	4
77	2	94	
92		97	
88		...	...
Rank Sum			

Wilcoxon Rank Sum Test Computation Table

Factory 1		Factory 2	
Rate	Rank	Rate	Rank
71	1	85	
82	~~3~~ 3.5	82	~~4~~ 3.5
77	2	94	
92		97	
88		...	...
Rank Sum			

Wilcoxon Rank Sum Test Computation Table

Factory 1		Factory 2	
Rate	Rank	Rate	Rank
71	1	85	5
82	~~3~~ 3.5	82	~~4~~ 3.5
77	2	94	
92		97	
88		...	...
Rank Sum			

Wilcoxon Rank Sum Test Computation Table

Factory 1		Factory 2	
Rate	Rank	Rate	Rank
71	1	85	5
82	~~3~~ 3.5	82	~~4~~ 3.5
77	2	94	
92		97	
88	6	...	...
Rank Sum			

Wilcoxon Rank Sum Test Computation Table

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Rank Sum			

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88	6	...	...
Rank Sum			

Wilcoxon Rank Sum Test Computation Table

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77	2	94	8
92	7	97	9
88	6	...	...
Rank Sum			

Wilcoxon Rank Sum Test Computation Table

Factory 1		Factory 2	
Rate	Rank	Rate	Rank
71	1	85	5
82	~~3~~ 3.5	82	~~4~~ 3.5
77	2	94	8
92	7	97	9
88	6	...	...
Rank Sum	19.5		25.5

Wilcoxon Rank Sum Test Solution

- **H_0: Identical Distrib.**
- **H_a: Shifted Left or Right**
- **$n_1 = 5$ $n_2 = 4$**
- **Critical Value(s):**

Reject	Do Not Reject	Reject
12	28	Σ Ranks

Test Statistic:

$T_2 = 5 + 3.5 + 8 + 9 = 25.5$ (Smallest Sample)

Decision:

Wilcoxon Rank Sum Test Solution

- **H_0: Identical Distrib.**
- **H_a: Shifted Left or Right**
- **$n_1 = 5$ $n_2 = 4$**
- **Critical Value(s):**

Reject	Do Not Reject	Reject

12 28 Σ Ranks

Test Statistic:

$T_2 = 5 + 3.5 + 8 + 9 = 25.5$ (Smallest Sample)

Decision:

Do Not Reject

Wilcoxon Rank Sum Test Solution

- H_0: Identical Distrib.
- H_a: Shifted Left or Right
- $n_1 = 6$ $n_2 = 5$
- Critical Value(s):

Reject	Do Not Reject	Reject
12	28	Σ Ranks

Test Statistic:

$T_2 = 5 + 3.5 + 8 + 9 = 25.5$ (Smallest Sample)

Decision:

Do Not Reject

Wilcoxon Rank Sum Test Activity

- For Team A, its population is composed of the following: **2**, **6**, **4**, **23**, **7**, **& 6**. For Team B, it has **6**, **8**, **7**, **10**, **& 8**. Do the populations of both have the same **probability distributions**?

Wilcoxon Rank Sum Test Computation Table

Team A		Team B	
Population	Rank	Population	Rank
2	1	6	4
6	4	8	8.5
4	2	7	6.5
23	11	10	10
7	6.5	8	8.5
6	4		
Rank Sum	28.5		37.5

Wilcoxon Rank Sum Test Solution

- **H_0: Identical Distrib.**
- **H_a: Shifted Left or Right**
- **$n_1 = 6$ $n_2 = 5$**
- **Critical Value(s):**

Reject	Do Not Reject	Reject
19	41	Σ Ranks

Test Statistic:

T_2 = 4+8.5+6.5+10+8.5 = 37.5 (Smallest Sample)

Decision:

Do Not Reject