

**KENDALL'S TAU  
CORRELATION BETWEEN  
RANKS**

# What is Kendall's ( $\tau$ ) Tau?

- It is a non-parametric measure of correlation between two ranked variables.
- Range: -1.0 to 1.0

## Formula: No Ties

$$\tau = \frac{P - Q}{n(n - 1)/2}$$

Wherein:

$\tau$  = Tau

P = no. of Concordant pairs

Q = no. of Discordant pairs

n = no. of pairs

# Example

# Raw Data of 7 students in 2 exams

## Kendall's Tau

Individual	Exam x	Exam y	$R_x$	$R_y$	Concordant	Discordant
Rylan	18	24				
Xavier	17	28				
Malik	14	30				
Greyson	13	26				
Landon	12	22				
Kaden	10	18				
Jasper	8	15				

Arrange data according to ranks in x in ascending order. Then pair the rank data in y with  $x_i$ .

## Kendall's Tau

Individual	Exam x	Exam y	$R_x$	$R_y$	Concordant	Discordant
Rylan	18	24	1	4		
Xavier	17	28	2	2		
Malik	14	30	3	1		
Greyson	13	26	4	3		
Landon	12	22	5	5		
Kaden	10	18	6	6		
Jasper	8	15	7	7		

Get the number of concordant and discordant pair.

# How to calculate?

**Concordant Pairs:** the number of observed ranks below a particular rank which are larger than that particular rank.

**Discordant Pairs:** the number of observed ranks below a particular rank which are smaller in value than that particular rank.

## Kendall's Tau

Individual	Exam x	Exam y	$R_x$	$R_y$	Concordant	Discordant
Rylan	18	24	1	4		
Xavier	17	28	2	2		
Malik	14	30	3	1		
Greyson	13	26	4	3		
Landon	12	22	5	5		
Kaden	10	18	6	6		
Jasper	8	15	7	7		



## Kendall's Tau

Individual	Exam x	Exam y	$R_x$	$R_y$	Concordant	Discordant
Rylan	18	24	1	4	3	
Xavier	17	28	2	2	4	
Malik	14	30	3	1	4	
Greyson	13	26	4	3	3	
Landon	12	22	5	5	2	
Kaden	10	18	6	6	1	
Jasper	8	15	7	7	0	

## Kendall's Tau

Individual	Exam x	Exam y	$R_x$	$R_y$	Concordant	Discordant
Rylan	18	24	1	4	3	3
Xavier	17	28	2	2	4	1
Malik	14	30	3	1	4	0
Greyson	13	26	4	3	3	0
Landon	12	22	5	5	2	0
Kaden	10	18	6	6	1	0
Jasper	8	15	7	7	0	0

## Kendall's Tau

Individual	Exam x	Exam y	$R_x$	$R_y$	Concordant	Discordant
Rylan	18	24	1	4	3	3
Xavier	17	28	2	2	4	1
Malik	14	30	3	1	4	0
Greyson	13	26	4	3	3	0
Landon	12	22	5	5	2	0
Kaden	10	18	6	6	1	0
Jasper	8	15	7	7	0	0
					P =	Q =

## Kendall's Tau

Individual	Exam x	Exam y	$R_x$	$R_y$	Concordant	Discordant
Rylan	18	24	1	4	3	3
Xavier	17	28	2	2	4	1
Malik	14	30	3	1	4	0
Greyson	13	26	4	3	3	0
Landon	12	22	5	5	2	0
Kaden	10	18	6	6	1	0
Jasper	8	15	7	7	0	0
					P = 17	Q = 4

**Solution:**

$$\tau = \frac{P - Q}{n(n - 1)/2}$$

$$= \frac{17 - 4}{7(7 - 1)/2}$$

$$= \frac{13}{21}$$

$$\tau = .62$$

# Formula: With Ties

$$\tau = \frac{P - Q}{\sqrt{(N - T_x)(N - T_y)}}$$

Wherein:

$\tau$  = Tau

P = total no. of  
Concordant pairs

Q = total no. of  
Discordant pairs

N = no. of comparison

$T_x$  = no. of consecutive  
ranks within x

$T_y$  = no. of consecutive  
ranks within y

To compute for T:

$$T_x = \frac{\sum n_{T_x}}{2}$$

$$T_y = \frac{\sum n_{T_y}}{2}$$

Wherein:

T = no. of consecutive ranks within x or y

$n_T$  = no. of sets of ties in the rank

$$n = x_n(x_n - 1)$$

x = no. of ties in a particular rank

# Example



Individual	Exam x	Exam y	$R_x$	$R_y$	Concordant	Discordant
Rylan	60	60				
Xavier	54	68				
Malik	53	40				
Greyson	49	52				
Landon	49	51				
Kaden	47	38				
Jasper	46	51				
Keith	45	32				
Angelo	45	39				
Nathan	45	41				
					<b>P =</b>	<b>Q =</b>

Individual	Exam x	Exam y	$R_x$	$R_y$	Concordant	Discordant
Rylan	60	60	1	2		
Xavier	54	68	2	1		
Malik	53	40	3	7		
Greyson	49	52	4.5	3		
Landon	49	51	4.5	4.5		
Kaden	47	38	6	9		
Jasper	46	51	7	4.5		
Keith	45	32	9	10		
Angelo	45	39	9	8		
Nathan	45	41	9	6		
					<b>P =</b>	<b>Q =</b>

Individual	Exam x	Exam y	$R_x$	$R_y$	Concordant	Discordant
Rylan	60	60	1	2	8	
Xavier	54	68	2	1	8	
Malik	53	40	3	7	3	
Greyson	49	52	4.5	3	6	
Landon	49	51	4.5	4.5	4	
Kaden	47	38	6	9	1	
Jasper	46	51	7	4.5	3	
Keith	45	32	9	10	0	
Angelo	45	39	9	8	0	
Nathan	45	41	9	6	0	
					<b>P = 33</b>	<b>Q =</b>

Individual	Exam x	Exam y	$R_x$	$R_y$	Concordant	Discordant
Rylan	60	60	1	2	8	1
Xavier	54	68	2	1	8	0
Malik	53	40	3	7	3	4
Greyson	49	52	4.5	3	6	0
Landon	49	51	4.5	4.5	4	0
Kaden	47	38	6	9	1	3
Jasper	46	51	7	4.5	3	0
Keith	45	32	9	10	0	2
Angelo	45	39	9	8	0	1
Nathan	45	41	9	6	0	0
					<b>P = 33</b>	<b>Q = 11</b>

# Solution: Solve for $T_x$ , $T_y$ and $N$

Kendall's Tau

$$\begin{aligned}T_x &= \frac{\sum n_{T_x}}{2} \\&= \frac{2(2 - 1) + 3(3 - 1)}{2} \\&= \frac{2 + 6}{2} \\&= \frac{8}{2} \\T_x &= 4\end{aligned}$$

# Solution: Solve for $T_x$ , $T_y$ and $N$

Kendall's Tau

$$\begin{aligned}T_y &= \frac{\sum n_{t_y}}{2} \\ &= \frac{2(2 - 1)}{2} \\ &= \frac{2}{2} \\ T_y &= 1\end{aligned}$$

$$\begin{aligned}N &= n(n - 1)/2 \\ &= \frac{10(10 - 1)}{2} \\ &= \frac{10(9)}{2} \\ &= \frac{90}{2} \\ N &= 45\end{aligned}$$

# Solution: Solve for $\tau$

## Kendall's Tau

$$\tau = \frac{P - Q}{\sqrt{(N - T_x)(N - T_y)}}$$

$$= \frac{33 - 11}{\sqrt{(45 - 4)(45 - 1)}}$$

$$= \frac{22}{\sqrt{(41)(44)}}$$

$$= \frac{22}{\sqrt{1804}}$$

$$= \frac{22}{42.474}$$

$$\tau = 0.52$$

# Test for Significance

$$Z = \frac{3\tau\sqrt{n(n-1)}}{\sqrt{2(2n+5)}}$$



Example 1:  $\tau = .62$

Kendall's Tau

$$Z = \frac{3\tau\sqrt{n(n-1)}}{\sqrt{2(2n+5)}}$$

$$Z = 1.96$$

$$= \frac{3(.62)\sqrt{7(7-1)}}{\sqrt{2(2(7)+5)}}$$

$$= \frac{1.86\sqrt{42}}{\sqrt{2(19)}}$$

$$= \frac{12.0541}{6.1644}$$

$$= 1.96$$

$$= 1.96$$

Example 2:  $\tau = .52$

Kendall's Tau

$$\begin{aligned} Z &= \frac{3\tau\sqrt{n(n-1)}}{\sqrt{2(2n+5)}} \\ &= \frac{3(.52)\sqrt{10(10-1)}}{\sqrt{2(2(10)+5)}} \\ &= \frac{1.56\sqrt{90}}{\sqrt{2(25)}} \\ &= \frac{14.7995}{7.0711} \end{aligned}$$

$$Z = 2.09$$

# Kendall's Tau

n \ $\alpha$							n \ $\alpha$						
	0.2	0.1	0.05	0.02	0.01	0.002		0.2	0.1	0.05	0.02	0.01	0.002
4	1.000	1.000	–	–	–	–	18	0.242	0.294	0.346	0.412	0.451	0.525
5	0.800	0.800	1.000	1.000	–	–	19	0.228	0.287	0.333	0.392	0.439	0.509
6	0.600	0.733	0.867	0.867	1.000	–	20	0.221	0.274	0.326	0.379	0.421	0.495
7	0.524	0.619	0.714	0.810	0.905	1.000	21	0.210	0.267	0.314	0.371	0.410	0.486
8	0.429	0.571	0.643	0.714	0.786	0.857	22	0.203	0.264	0.307	0.359	0.394	0.472
9	0.389	0.500	0.556	0.667	0.722	0.833	23	0.202	0.257	0.296	0.352	0.391	0.455
10	0.378	0.467	0.551	0.600	0.644	0.778	24	0.196	0.246	0.290	0.341	0.377	0.449
11	0.345	0.418	0.491	0.564	0.600	0.709	25	0.193	0.240	0.287	0.333	0.367	0.440
12	0.303	0.394	0.455	0.545	0.576	0.667	26	0.188	0.237	0.280	0.329	0.360	0.428
13	0.308	0.359	0.436	0.513	0.564	0.641	27	0.179	0.231	0.271	0.322	0.356	0.419
14	0.275	0.363	0.407	0.473	0.516	0.604	28	0.180	0.228	0.265	0.312	0.344	0.413
15	0.276	0.333	0.390	0.467	0.505	0.581	29	0.172	0.222	0.261	0.310	0.340	0.404
16	0.250	0.317	0.383	0.433	0.483	0.567	30	0.172	0.218	0.255	0.301	0.333	0.393
17	0.250	0.309	0.368	0.426	0.471	0.544							

Thank you!

*"That in All Things, God maybe Glorified!"*